Occlusion Vehicle Detection Algorithm in Crowded Scene for Traffic Surveillance System

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***Abstract*— Traffic Surveillance System (TSS) plays an important role in extracting necessary information (count, type, speed, etc.). In the area of Traffic Surveillance System (TSS), vehicle detection has emerged as an influential field of study. So far there has been a considerable amount of research to accommodate this subject. However, these studies almost address problems in developed countries where the traffic infrastructure is constructed to appropriate automobiles. Detecting moving vehicles in urban areas is difficult because the inter-vehicle space is significantly reduced, increasing the occlusion between vehicles. This issue is more challenging in developing countries where the roads are crowded with 2-wheeled motorbikes in rush hours. This paper presents a method to improve the occlusion vehicle detection from static surveillance cameras. The proposed method is a vision-based approach in which undefined blobs of occluded vehicles are examined to extract the vehicles individually based on the geometric and the ellipticity characteristic of objects' shapes. Experiments have been carried out with the real-world data to evaluate the performance and the accuracy of our method. The assessment results are promising for a detection rate of 84.10% at daytime.**

***Keywords*— occlusion detection, blob splitting, vehicle segmentation, traffic surveillance system**

I. Introduction

The past decade has seen the explosion of intelligent and expert systems, especially in the area of transportation management. Traffic surveillance system (TSS) has gained popularity among researchers and authorities. A key aspect of TSS is to derive the traffic information (count, average speed, and the density of each vehicle type) for further analysis related to traffic management and planning. In this context, many studies have been conducted in developed countries where the transportation frameworks are constructed primarily for automobiles. These systems [1], [2] were developed with the advanced equipment and sensors to optimize the incoming signal including radar, infrared camera and so on. However, in developing countries, the application of these systems has trouble with high cost and incompatible infrastructures. On the contrary, the vision-based TSSs which are built from computer vision and image processing techniques [3]–[5] have shown more superior capability with lower cost and easier to maintain. Moreover, they are extremely versatile as algorithms are designed to cope with a broad range of operations such as detect, identify, count, track, and classify vehicles.

In vision-based TSS, vehicle detection is one of the most important operation since it plays a major role in localizing moving vehicles in traffic video. This vital tool is a well-defined and has been studied in a considerable amount of works [6]. An early research which carried out by Buch et al. [7], [8] proposed motion silhouettes or 3D Histograms of Oriented Gradients (3DHOG) model that is used to localize and to classify a variety of vehicles and pedestrians with a noticeable precision but the resolution for occlusion is still inconsiderable. Chunyu and Shihui [9] demonstrate a method which uses skeleton features to segment the occlusion blobs of medium (cars, van, sedan) and heavy (trucks, buses) vehicles with good accuracy. However, with the appearance of motorized two-wheelers whose shapes vary substantially and have smaller sizes, it is a great challenge for these existing studies to locate the segmentation points because of the complicated presence of occlusion among motorbikes. Particularly, Chi and Thanh et al. [10], [11] are one of a few whose studies concentrate on motorbike detection in developing countries by establishing a convolutional neural network (CNN) and a Bag-of-Words (BoW) model. These models are tolerated to perform in dense traffic, but their works have to suffer a long computational time for training phrase and are impossible to integrate into real-time system despite the assistance of graphics processing unit (GPU). Notably, some recent works by Ha and Pham et al. [5], [12] have shown remarkable outcomes in detecting and classifying vehicles in urban areas. However, urban vehicle detection has its challenges that cannot be easily addressed by these methods due to the occurrence of occlusions. The problem is more challenging in rush hour when the traffic is slower, and the inter-vehicle space is significantly reduced which increases the occlusion between vehicles. Adding to this is the immense density of motorbikes which is the main cause for chaos on urban roads in Vietnam. Motorbikes have non-rigid shapes, and their appearances vary widely, in particular, when their pose changes as they move throughout the scene.

In this paper, we propose a robust vehicle detection algorithm that handles the occlusions of 2-wheeled motorized vehicles in crowded traffic scenes. Our work is an extension of [5], [12] which proposed a robust TSS comprising of three main components: background subtraction, vehicle detection, and classification. As shown in Fig. 1, the main contribution in our paper is introducing an occlusion detection method and an overlapping vehicle segmentation algorithm which has been developed with a data-driven approach on real-world data. Like previous studies, we use background subtraction to model the background, from which moving vehicles can be detected. Then blobs of overlapping vehicles are identified

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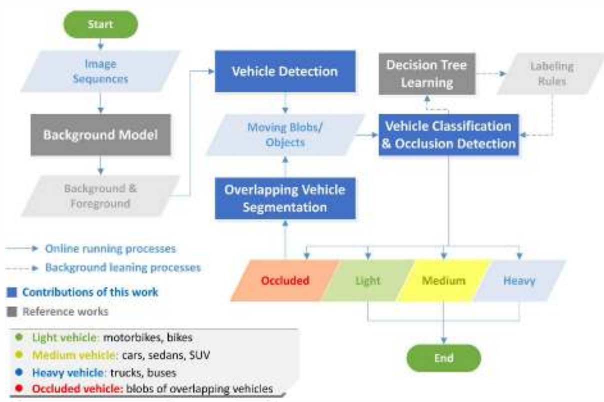


Fig. 1. The workflow of our proposed traffic surveillance system.

based on the geometric characteristics and the features of objects' shapes. This assessment is done using a decision tree constructed from a training set of 10,000 real-world vehicle images captured in Ho Chi Minh City, Vietnam. Once occluded vehicles are extracted, we proceed with the overlapping vehicles segmentation process. We propose a novel segmentation method that performs exhaustive checking and pairing of defect points in the object contours. The blobs resulted from each cut are validated with the vehicle model [5] consisting of vehicle size S, dimension ratio R_{di}, density ratio R_{de}, and ellipticity characteristics. Experiments have shown promising results with high vehicle detection rates of 84% on the considered data.

II. The Proposed Method

This paper presents an occlusion vehicle detection method in the way of integrating four modules: background subtraction, vehicle detection, occlusion detection and overlapping vehicle segmentation.

A. Background Subtraction

In our approach, initially, background subtraction model is utilized to define two main parts of the traffic scene: moving foregrounds and static backgrounds. To be more specific, the foregrounds are marked as the white blobs, and the backgrounds are indicated by black regions in binary images. However, traffic images received through a digital camera are not always stable and usually experience in unfavorable conditions such as the illumination change, the presence of motionless vehicles, and camera vibration in the outdoor environment. To overcome these problems, the background modeling proposed by Nguyen et al. [3] is undertaken radically. Fig. 2(a)-(c) illustrates the background construction and the background subtraction procedure.

B. Vehicle detection

After constructing a stable background model, the foreground which contains moving objects is extracted. In addition, by narrowing down the examining area, we eliminate the unexpected objects as well as reduce the number of blobs that we need to investigate. Regarding to this issue, we adopt the construction of vehicle detection

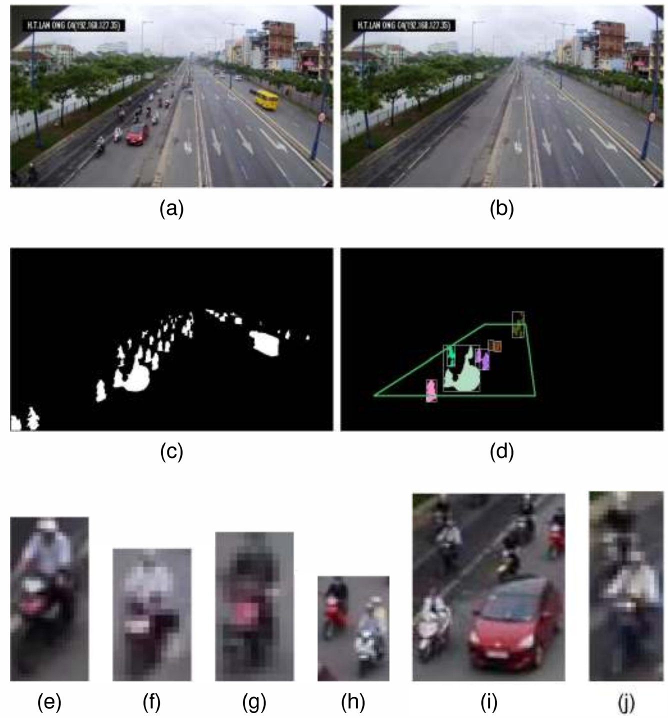


Fig. 2. (a) Original image. (b) Background image. (c) Foreground image from background subtraction process. (d) Extracted blobs of moving objects inside the examining area. (e)-(j) Extracted blob images.

which was proposed by Ha and Pham et al. [5], [12], [13]. Literally, the contour of each moving candidate is determined by applying the technique that is stated clearly in [14]. Fig. 2(d) exemplifies the result of this manner. Following with this task, the border that defines object individually is bounded with ellipse so that the low-level features are elicited sustainably without regarding the fluctuation of vehicles' appearances [15]. For each blob detected, we obtain a collection of geometric features as it moves throughout the scene. The characteristics of ith candidate at tth frame are outlined and illustrated in Table I and Fig. 3.

TABLE I
VEHICLE'S MEASUREMENT

Symbol	Description
$E_i(t)$	Vehicle's bounding ellipse
$\theta_i^E(t)$	Ellipses rotation angle
$E_i^W(t)$	Bounding ellipses width
$E_i^H(t)$	Bounding ellipses height
$P_i^E(t)$	Bounding ellipse size
$P_i^C(t)$	Vehicle size

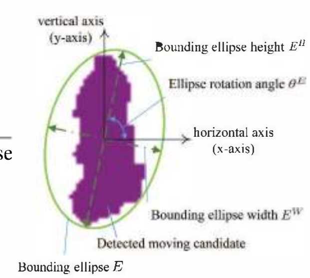
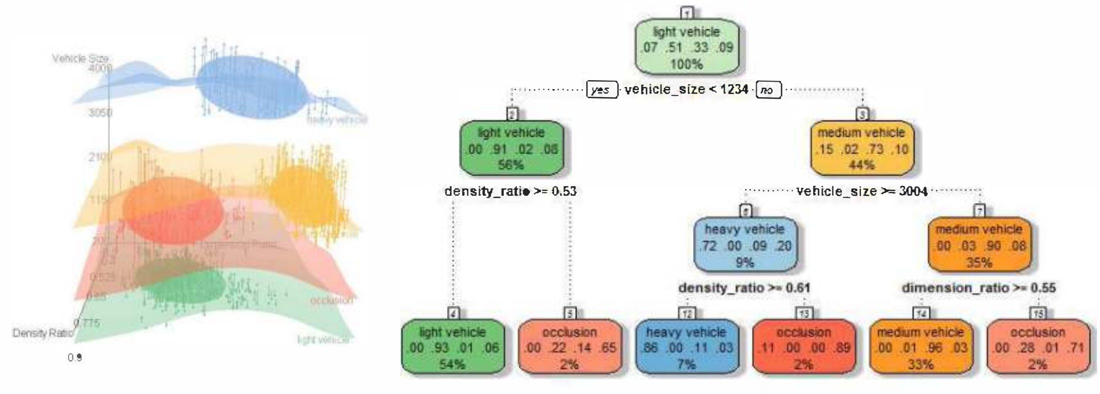


Fig. 3. Ellipticity properties

A major advantage of vehicle detection which is mentioned in this section is to detect separate blobs of moving vehicles with the remarkably small amount of time, and the issue of duplicate counting is almost resolved. However, in the circumstance of occlusion, there is a severe deficiency in vehicles detection, which is caused by a compound blend of the individual candidates and the clusters of overlapping



(a) (b)

Fig. 4. (a) The scatter plot of vehicles from dataset VVK1. (b) The decision tree categorizing the vehicles and occlusion part in dataset VVK1.

conveyances in the detected set as shown in Fig. 2(e)-(j). Nevertheless, little consideration has been made in the scene where the inter-distance between vehicles reduces notably.

C. Occlusion Detection

Regarding the current investigation, overlapping vehicle segmentation which is our first main contribution in this paper is a mechanism that segregates blobs of overlapping objects from other kinds of candidates in the set detected from Section II-B into four classes:

- **Class 1: Light vehicle**, consisting of bike and motorbike.
- **Class 2: Medium vehicle**, consisting of car, sedan, and 12-seater bus.
- **Class 3: Heavy vehicle**, consisting of truck, trailer, 16-to-50-seater bus.
- **Class 4: Occluded vehicle**, which is the blob of overlapping vehicles.

Following the essence of vehicle classification, we construct an evaluation procedure generating a tuple of three informative geometric features that depict the four kinds of vehicle blobs. The first assessment is vehicle size P^C, the total number of pixels bounded by vehicle's contour. The second measurement is density ratio R^{de} achieved by calculating the proportion of vehicle size to the bounding ellipse size P^E. The last appraisal is dimension ratio R^{di} of bounding ellipse width E^W to height E^H:

$$R_i^{di}(t) = \frac{E_i^W(t)}{E_i^H(t)}; R_i^{de}(t) = \frac{P_i^C(t)}{P_i^E(t)} \tag{1}$$

According to the above literature, a considerable amount of investigations have been performed on collected data. To be more specific, the detected blobs which are the outputs from vehicle detection module are manually marked with appropriate labels indicating the corresponding categories of vehicles in order to prepare for analysis process. Fig. 4(a) presents a scatter plot which illustrates the distribution of vehicles features in 3D space containing three axes that signify three defined attributes (P^C, R^{di}, R^{de}). In this figure, the green, yellow, blue, and red dots respectively describe light, medium, heavy, and occluded vehicles. A significant observation from this scatter diagram is that there is a substantial separation in dimension ratio among these groups. In particular, R^{di} of automobiles is greater than the value of motorbikes. The reason for this matter is that at field of view of surveillance cameras, the height of the medium and the heavy vehicles appears to be shorter than the one in reality. Towards motorized two-wheelers, there is a disparity between the vertical and horizontal linear measurement. Moreover, the height of blobs in this class is also affected by the presence of motorists. However, because of the complication of the state of occlusion, the dimensions of blobs of obscured vehicles remarkably vary, which leads to the confusion of distinguishing with the other classes. At this point, density ratio R^{de} is a noteworthy investigation to resolve this problem. Obviously, the portion of inner spaces among occluded candidates form the gaps interleaving in blobs detected. For that reason, the density ratio of occlusion blob is the least when comparing with three other categories of conveyances. Nevertheless, borderline candidates are still indecisive to be distributed to a class of vehicle. Under those circumstances, by inspecting the vehicle size P^C, four groups are obviously separated by surfaces which are parallel to the plane (R^{di}, R^{de}).

From this observation, a set of tuples (P^C, R^{di}, R^{de}) which are extracted from an amount of data is utilized to construct a categorizing structure. In this research, we adopt decision tree approach that is presented by [16] to build up a classification model. In brief, decision tree is a method classifying a batch of discrete samples. With respective to this model, the input data is distributed on an architectural tree with a group of nodes consisting of a root node, some internal nodes, and several leaf nodes. This structure can be obtained by using a top-down, greedy search algorithm, especially ID3, which focuses on finding out the best classifiers among the candidates attributes to disassemble a large collection into smaller identified groups via a statistical assessment. Fig. 4(b) presents the decision tree that corresponds to the data VVK1. Starting at the root node, on the left branch, the light vehicles and the occlusion blobs share the same size criteria, but they distinguish each other with the specification

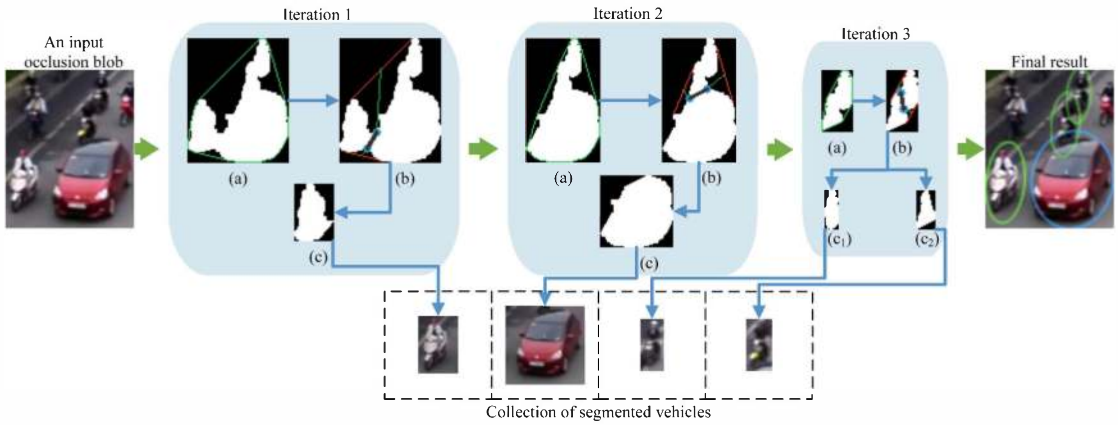


Fig. 5. The overlapping vehicle segmentation. (a) Convex bounding polygon computation. (b) Concave spot localization. (c) Segmented individual vehicles.

of the density ratio. On the contrary, at the right branch, the remaining candidates are distributed with a particular vehicle size evaluation. Continuously, the smaller subset finally completes the compartmentalization through appropriate constraints on the density and the dimension ratio. Apparently, with this model, the blobs of obscured vehicles are drawn out after the detection phrase as a preparation for overlapping vehicle segmentation.

D. Overlapping Vehicle Segmentation

Once the occlusion blobs of vehicles are categorized as exceptional instances of experimental subjects, at this stage, we continue to examine the bundle segmentation of detected candidates into the individual vehicles. Dealing with this issue, we present the second contribution that is a robust solution to handle the overlapping vehicle segmentation.

As stated earlier in the introduction of this research, our proposed method is a video-based manner that processes directly on a sequence of captured images from surveillance cameras. Undeniably, pictorial signals received through the system can be affected by a variety of external factors including camera vibration, illumination change, and interference from other devices. For these reasons, before initiating our principal procedure, the received data which is the set of occlusion blobs designated from Section II-C has to undergo a preliminary convention. As the foreground blobs are presented in binary image, in order to get rid of jagged edges and to refine the bounding contours of objects, morphology operators is appropriate solutions in this phase of pre-processing.

In our approach, the occlusion segmentation is a repetitive process which partition the obscured objects alternately. In normal conditions, regardless of small defects, the appearances of individual vehicles are considered as curving outward shapes. Therefore, in case of overlapping among vehicles, there are concave spots on the edge of detected blob, which is a corollary of the overlapping phenomena among moving conveyances. These positions are effective indications for blob splitting. Hence, in the first step of this method, we construct a convex bounding polygon ρ for the border of each detected blob μ_k that comprising a set of k vertices p_i, which is mathematically indicated as:

$$\rho = \left\{ \sum_{i=1}^{k} \lambda_i p_i | p_i \in \mu_k \wedge \lambda_i \geq 0 \wedge \sum_{i=1}^{k} \lambda_i = 1 \right\} \quad (2)$$

In this matter, an optimal algorithm is thoroughly presented by Sklansky [17]. Step (a) at each iteration in Fig. 5 illustrates the convex bounding polygon of occlusion blob. From that outcome, we continue to determine potential points for later process. In particular, this set σ_m consists of m concave spots which are on the outer boundary of examining objects but do not belong to the polygon ρ. Step (b) shows two selected concave spots that are utilized to segment the occlusion blob. In this figure, two attributes are presented to describe the importance of a detected locality. The first characteristic, denoted by a red line segment, is defect width that is the length of the bounding convex polygon's edge of inspecting spot. The other property, indicated as a green line, is defect depth which is distance from the concave spot to the midpoint of the corresponding bounding convex polygon's edge. Furthermore, in this investigation, to select correct segment points and to eliminate unnecessary impurities, two constraints attained from practical experiments are considered on detected set σ_m:

1) The defect width must be greater than certain threshold.

$$\|\rho_i - \rho_j\| \geq TH_1 \text{ where } \rho_i, \rho_j \in \rho \quad (3)$$

where TH_1 is the minimum defect width and set to 7 pixels.

2) The defect depth is limited with an interval bounded by two thresholds.

$$TH_2 \leq \left\| \sigma_t - \left(\frac{\rho_i + \rho_j}{2} \right) \right\| \leq TH_3 \quad (4)$$
$$\text{where } \rho_i, \rho_j \in \rho \text{ and } \sigma_t \in \sigma_m$$

where TH_2 and TH_3 are respectively the minimum and the maximum defect depth that are alternately set to 3 pixels and 30 pixels.

Afterwards, for each pair of defined points, we form a cutting line which is utilized to separate obscured vehicles as step (c) at each iteration in Fig. 5. Subsequently, the

TABLE II
COMPARISON OF RESULTS BETWEEN HA'S METHOD AND OUR PROPOSED METHOD.

			Ha's method [5]				Our method			
Dataset	Class	Actual	Detect	Deviation	Percent	T.Acc	Detect	Deviation	**Percent**	**T.Acc**
HMD01	1	498	250	248	50.20%	54.06%	427	71	**85.74%**	**84.60%**
	2	68	47	21	69.12%		56	12	**82.35%**	
	3	7	3	4	42.86%		6	1	**85.71%**	
NTL01	1	652	364	288	55.83%	62.54%	538	114	**82.52%**	**84.97%**
	2	168	115	26	68.45%		144	12	**85.71%**	
	3	30	19	2	63.33%		26	4	**86.67%**	
COL01	1	574	392	182	68.30%	66.97%	483	91	**84.15%**	**82.70%**
	2	32	21	11	65.63%		26	6	**81.25%**	
	3	0	0	0	-		0	0	-	
Overall Accuracy			61.19%				**84.09%**			

segmented candidate after reconstructing the necessary vehicle's measurement is verified through the state decision tree mentioned in Section II-C. This manner takes 2 to 17 iterations of processes to attain the convergence of results. The procedure ends at the stage when all individual vehicles are identified. The rightmost image in Fig. 5 shows the final result of segmentation procedure.

III. EXPERIMENTS AND DISCUSSION

Several experiments have been performed on the selected data to evaluate the proposed method. In these examinations, the testing datasets are captured from static pole-mounted surveillance cameras in Ho Chi Minh City at the rate of 30 fps with a resolution of standard VGA (640x480) to assess the accuracy and to test the performance of our method during crowded scenes. Technically, these cameras are set up at the height of $8-9$ meter and inclined at an angle of $12^o - 15^o$ to the horizontal direction. In addition to this, the system which is utilized for these investigations has a configuration of Intel Core i7 2630QM and 8GB of RAM.

In the previous studies, Ha and Pham et al. demonstrated a robust algorithm to detect and to classify different kinds of vehicles in daytime surveillance environment with a remarkable accomplishment [5], [12]. In this paper, we continue to improve prior attainments by initiating a novel method to proceed the occlusion in vehicle detection. Table II summarizes the results of the previous study which is presented by Ha [5] and our proposed method. Both of two solutions are examined on three different datasets HMD01, NTL01, and COL01 which depict the different levels of occlusion. Dataset HMD01 was captured on the highway during rush hour in the morning. In this dataset, because there is lane separation between motorbike and automobile, two approaches mainly concentrate on detecting and classifying overlapping vehicles of the same group. Also, in order to extend our investigation, dataset NTL01, which describes the high density of vehicles in the residential area, is considered as an appropriate sample to assess a complicated circumstance with mixed-flow lanes where three classes of vehicles mix together. Last but not least, dataset COL01 plays a major role in scrutinizing the higher level of occlusion among the light and the medium conveyances. Fig. 6 presents some examples of three experiments.

As shown in Table II, the results obtained from the analysis of two methods on different datasets show that our algorithm improves the overlapping vehicle detection and accurately classifies the segmented candidates into three class with confidence up to 85%. When comparing with Ha's method whose studies are most relevant to ours, in intricate circumstances where there is a considerable presence of obscured vehicle, our overlapping vehicle segmentation algorithm significantly enhances the capability of TSS by 20% and increases adaptation in complicated situations. The proposed method can detect at least 82% of moving overlapping motorbikes and over 81% of medium vehicles. In particular, this result primarily depends greatly on the verification procedure as we mentioned in Section II-C. Moreover, our approach of overlapping vehicle segmentation relies much on the geometric characteristics and the ellipticity attributes of detected candidates on the whole. Hence, in some minor cases, unexpected objects are erroneously gathered as desired vehicles such as pedestrians, rudimentary means of transportation, and slabs of foreground faults caused by sudden illumination change in background subtraction model. In general, our method not only provides the result with high accuracy, but also retains the low deviation when classifying light, medium, and heavy vehicles. This result is only achieved by the contributing an effective vehicle detection, a steady verification model, and a robust segmentation algorithm.

TABLE III
AVERAGE COMPUTING TIME

Dataset	Total frames	Total run-time (sec.)	Frames per second (fps)
HMD01	9057	330.102	27.44
NTL01	9060	346.237	26.17
COL01	8820	334.421	26.37

Besides outperforming the outcome with an overall accuracy of roughly 84.1%, the proposed method maintains a small amount of computational time on the whole solution.

Fig. 6. Examples of occlusion vehicle detection and classification result with three experimental datasets. Green, yellow, blue ellipses respectively indicate light, medium, and heavy vehicles. The 1^{st}, the 2^{nd} and the 3^{rd} column respectively present results from dataset HMD01, NTL01 and COL01.

Table III sums up performance on each experiment. To be more specific, the tests were performed with a processing of approximately 9,000 frames in each execution. With the high density of occlusion which requires such more computations, the process rate is around 26 fps because the system needs to run extraordinary computation. On the contrary, with a lower degree of occlusion as in dataset HMD01, the frame rate can reach up to over 27 fps. Accordingly, our method process traffic data in real-time and possibly integrates into the existing TSS.

IV. Conclusion

This paper proposes a new method for overlapping vehicle segmentation to handle vehicle detection in circumstances of occlusion. The contribution of our research is a vision-based approach which utilizes the typical geometric features and the ellipticity characteristics to localize the conveyances inside the occlusion blob individually and to classify vehicles into 3 classes: light, medium, and heavy vehicle. In this investigation, the experiments on the suggested algorithm show some promising of results with the average accuracy over 84% and robust adaptability to the real-time performance at the overall frame rate of 27 fps.

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